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APPLICATION OF METAMODEL-ASSISTED MULTIPLE-GRADIENT DESCENT ALGORITHM (MGDA) TO AIR-COOLING DUCT SHAPE OPTIMIZATION

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Abstract. *MGDA stands for Multiple-Gradient Descent Algorithm was introduced in [1]. In a previous report [2], MGDA was tested on several analytical test cases and also compared with a well-known Evolution Strategy algorithm, Pareto Archived Evolution Strategy (PAES) [3]. Using MGDA in a multi-objective optimization problem requires the evaluation of a substantial number of points with regard to criteria, and their gradients. In industrial test cases, in which computing the objective functions is CPU demanding, a variant of the method was to be found. Here, a metamodel-assisted MGDA is proposed and tested. The MGDA is assisted by a Kriging surrogate model construction. A first database is computed as an Latin Hypercube Sampling (LHS) distribution in the admissible design space, which is problem-dependent. Then, MGDA leads each database point to a non dominated set of the surrogate model. In this way, each function computation is made on the surrogate model at a negligible computational cost.*

1 INTRODUCTION

MGDA stands for Multiple-Gradient Descent Algorithm [1]. It is a generalization of the classical steepest-descent method [4] that applies to cases in which an arbitrary number of criteria, of known gradients, are to minimized, or simply reduced. It is based on the observation that if ω is the minimal norm element in the convex hull of these gradients, then $-\omega$ is a descent direction for all criteria simultaneously. If $\omega = 0$, the considered point belongs to the Pareto set [1]. MGDA was tested successfully in [2] on several analytical test cases proposed in [5] to asses evolution strategies. One of then is reported here.
Using MGDA in a multi-objective optimization problem requires the evaluation of a substantial number of points with regard to criteria, and their gradients. In the particular case of Computational Fluid Dynamics (CFD) problems, each point evaluation is very costly, in term of CPU. Thus, here, we propose to alleviate this difficulty by constructing metamodels and calculating approximate gradients.
As a test case for numerical experiments, we consider the problem of shape optimization of an automobile air-cooling duct. It consists of the simultaneous minimization of two criteria associated with a compressible Navier-Stokes flow inside the duct.

2 MGDA

2.1 MGDA principles

MGDA is a gradient based method of multi-objective optimization. This method could be applied to cases in which an arbitrary number of criteria, of known gradients, are to be minimized. A particular point in the design space is called Pareto-optimal when it is non dominated every other acceptable point. Thus consider n smooth criteria $J_i(Y)$ (Y : design vector ; here in $\mathbb{R}^N$). For particular reasons explained in [1], $n \leq N$ and cost functions are assumed to be C^2 in some working ball in the design space $\mathbb{R}^N$. Let Y^0 be a Pareto-optimal point of the smooth criteria ($J_i(Y^0)$) and define the gradient vectors $u_i^0 = \nabla J_i(Y^0)$ in which ∇ denotes the gradient operator. There exists a convex combination of the gradient-vectors that is equal to zero. Inversely, if the smooth criteria $J_i(Y)$ are not Pareto-optimal at a given point Y, descent directions common to all criteria exist. Let $\overline{\mathcal{U}}$ (Equation 1) be the convex hull of the gradient vectors :

$$\overline{\mathcal{U}} = \left\{ w \in \mathbb{R}^N, \quad w = \sum_{i=1}^{n} \alpha_i u_i, \quad , \alpha_i \geq 0, \quad \sum_{i=1}^{n} \alpha_i = 1 \right\}. \tag{1}$$

$\overline{\mathcal{U}}$ is a closed and convex set. This implies existence and uniqueness of the element ω of minimum norm in $\overline{\mathcal{U}}$ (Equation 2) :

$$\omega = \min_{w \in \overline{\mathcal{U}}} \|w\|. \tag{2}$$

Then, $-\omega$ is a descent direction for all criteria simultaneously. If $\omega = 0$, the point is on the Pareto set. The particular case of two criteria is illustrated in Figure 1.

Assume $Y^0 \in \mathbb{R}^N$, a particular design vector and $J_i(Y^0)$ the i^{th} criteria value computed at this particular point. For each criterion, at Y^0, the gradient value been computed. Then, the minimal norm vector in the convex hull of the gradients is denoted ω. If $\|\omega\| > \varepsilon$, (ε is a user

[1]Actually a slightly lees stringent condition holds : Pareto-stationarity [1]

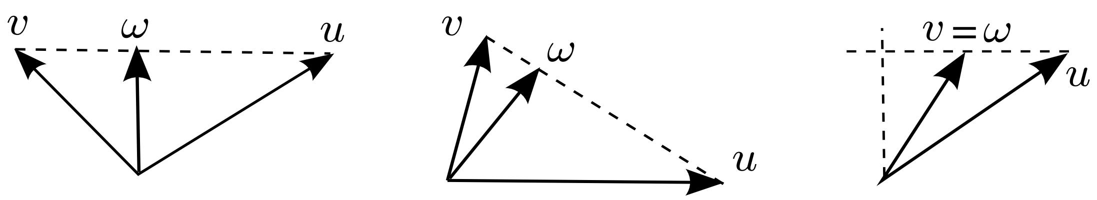


Figure 1: Assume u and v two vectors. In each case, ω represent the minimal norm vector of the $\{u, v\}$ convex hull.

fixed parameter), a new point in $-\omega$ direction which dominates (in Pareto sense) Y^0 exists. In this particular direction, an optimal step size ρ is computed as into by fitting a local problem explained in [2]. Then, the new point $Y^0 - \rho\omega$ is used as starting point by the MGDA. This method is summarized in Figure 2

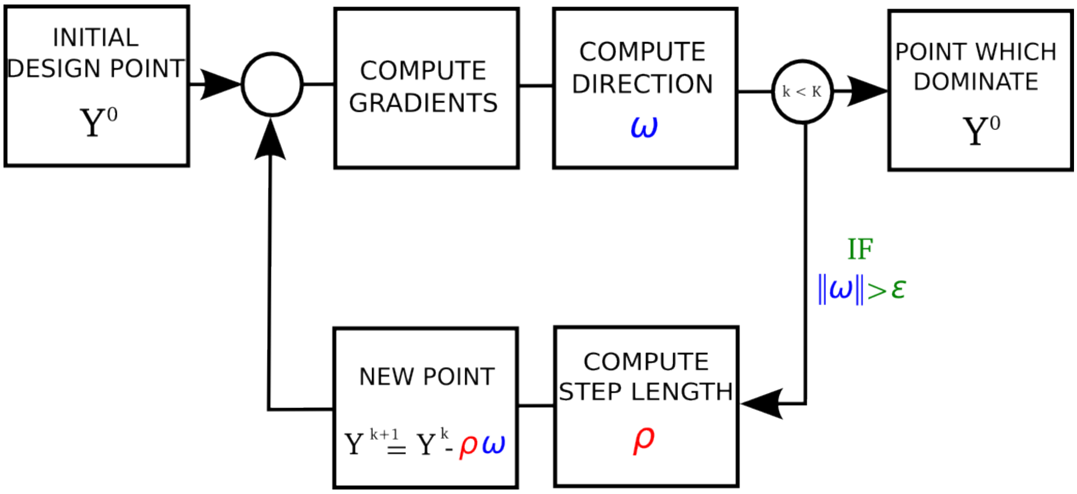


Figure 2: Modular scheme of the *Multiple Gradient Descent Algorithm* (MGDA)

2.2 Analytical validation

In this section, MGDA is illustrated on a problem proposed by Fonseca [5]. This test case corresponds to the two-objective unconstrained minimization of the following functions :

$$\begin{cases} f_1(Y) = 1 - \exp\left(-\sum_{i=1}^{3}\left(y_i - \frac{1}{\sqrt{3}}\right)^2\right) \\ f_2(Y) = 1 - \exp\left(-\sum_{i=1}^{3}\left(y_i + \frac{1}{\sqrt{3}}\right)^2\right) \end{cases}, \quad Y = (y_1, y_2, y_3). \tag{3}$$

The design variable is $Y = (y_1, y_2, y_3) \in \mathbb{R}^3$. This test case is known to yield a continuous but non convex Pareto set in function space. The Pareto front was identified by Deb using the well-known genetic algorithm NSGA-II [5]. Furthermore, the Pareto front can be calculated analytically.

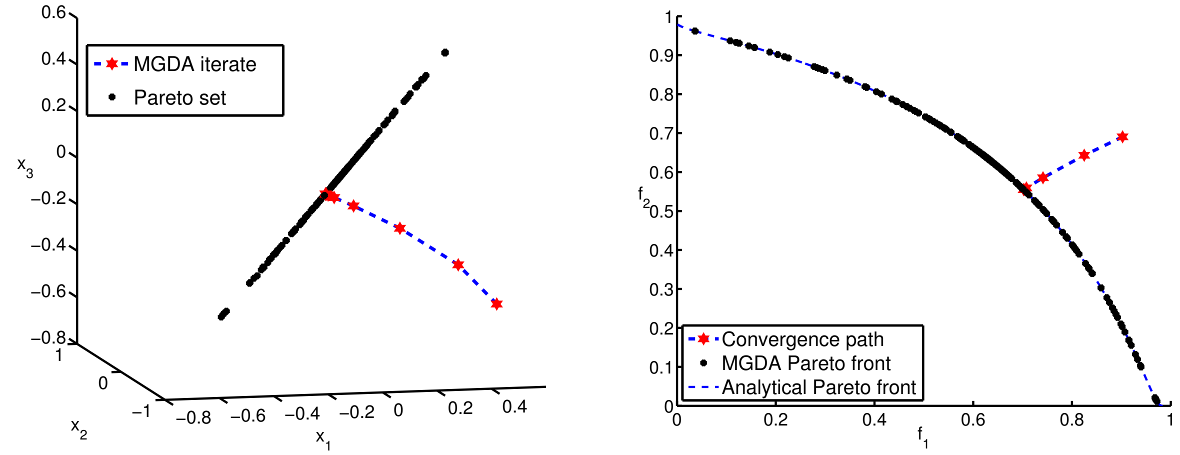


Figure 3: Convergence of MGDA from an initial design point to the non dominated set.

From a starting point, MGDA converges rapidly (6 steps in the example chosen) and provides an accurately defined point on the Pareto set Figure 3. A set of initial design points produces an accurate set of non dominated points.

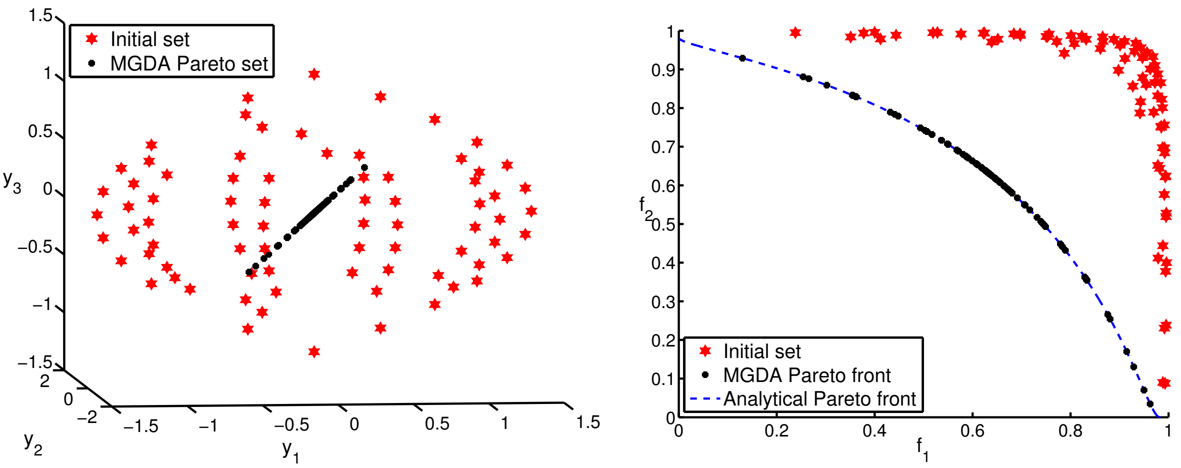


Figure 4: Convergence of MGDA from initial design points around Pareto set, for a classical test case proposed by Fonseca, in design space (left), in function space (right). In the functional space, the analytical Pareto front is plotted to compare.

To obtain an accurate representation of the Pareto set by MGDA, we have applied the method starting from a set of 60 initial design points distributed on a sphere in the design space, around the known non dominated set on Figure 4. The results are similar to those obtained by Deb using NSGA2.

In summary, MGDA converges rapidly from an initial design point to a non dominated point. At each iteration, the cost functions are evaluated twice at least in order to determine the step size. This remains possible for fast function evaluations, in term of computational time. In case of functions with an expensive computational cost, the strategy must be adapted.

3 Metamodel assisted MGDA

3.1 Metamodel assisted MGDA principles

Using MGDA in a multi-objective optimization problem requires the evaluation of a substantial number of points with regard to the criteria of interest along with their gradients. In the particular case of non-linear problems, such as CFD or non-linear mechanics problems, each point evaluation is very costly. Thus, we propose to overcome this difficulty by estimating function values on a surrogate model corresponding to the response of the functions within the design search space.

An initial set of design points is generated by using of a LHS method in $\mathbb{R}^N$. As illustrated in Figure 5, the databases are also used to provide starting points to initiate the MGDA iteration, which is conducted until convergence using gradients that are calculated on the basis of the metamodels. Each converged point belongs to the Pareto set associated with the two-criterion

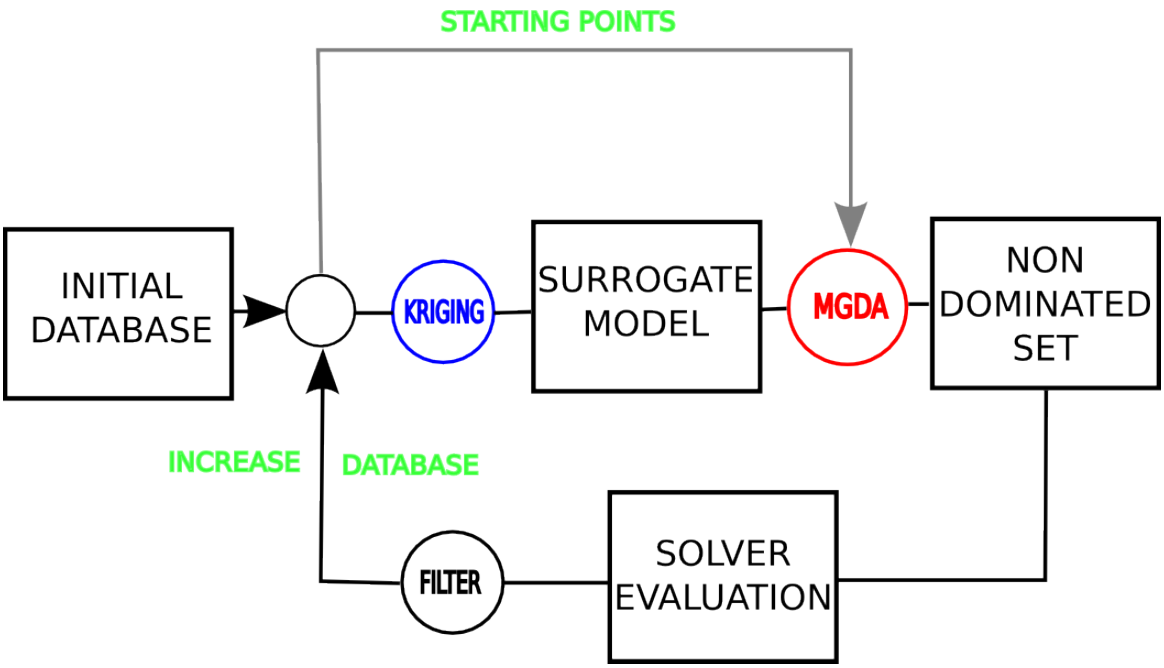


Figure 5: MGDA with surrogate model scheme. A surrogate model based on an initial database is trained. Then, MGDA [1] is applied from each database point. Thus a non dominated set on the surrogate model is obtained. Unless a non dominated point obtained is too close to an initial database element, its exact performance is computed and used to increase the database.

problem related to the metamodels. This point is then reevaluated by a flow computation, and added to the database. A filtering method is used to remove this point if it is too close to an already existing point. At completion of this database enrichment process, the metamodel is updated, which describes the cycle. For more robustness and to reduce the antagonism the conflict between parameters of different nature (angle, length, ...) and also different scales, during the optimization, all parameter intervals are scaled to be in $[-1,\, 1]$.

3.2 Numerical experimentation

As a test case for numerical experiments, we consider the problem of shape optimization of an air-cooling duct in which an compressible Navier-Stokes flow is considered. The inlet and outlet sections are fixed. Thus, only the shape of the elbow is optimized. The elbow between these two sections is defined by 8 parameters, 3 angles (X_2, X_5 and X_6), and 5 lengths (X_1, X_3, X_4, X_7 and X_8). See Figure 6. The shape is represented by 8 B-spline functions joining the inlet duct and the middle duct of the elbow. Eight more B-splines maintain a smooth junction between the middle part and the outlet. Each parameter can vary in the interval $[-1, 1]$.

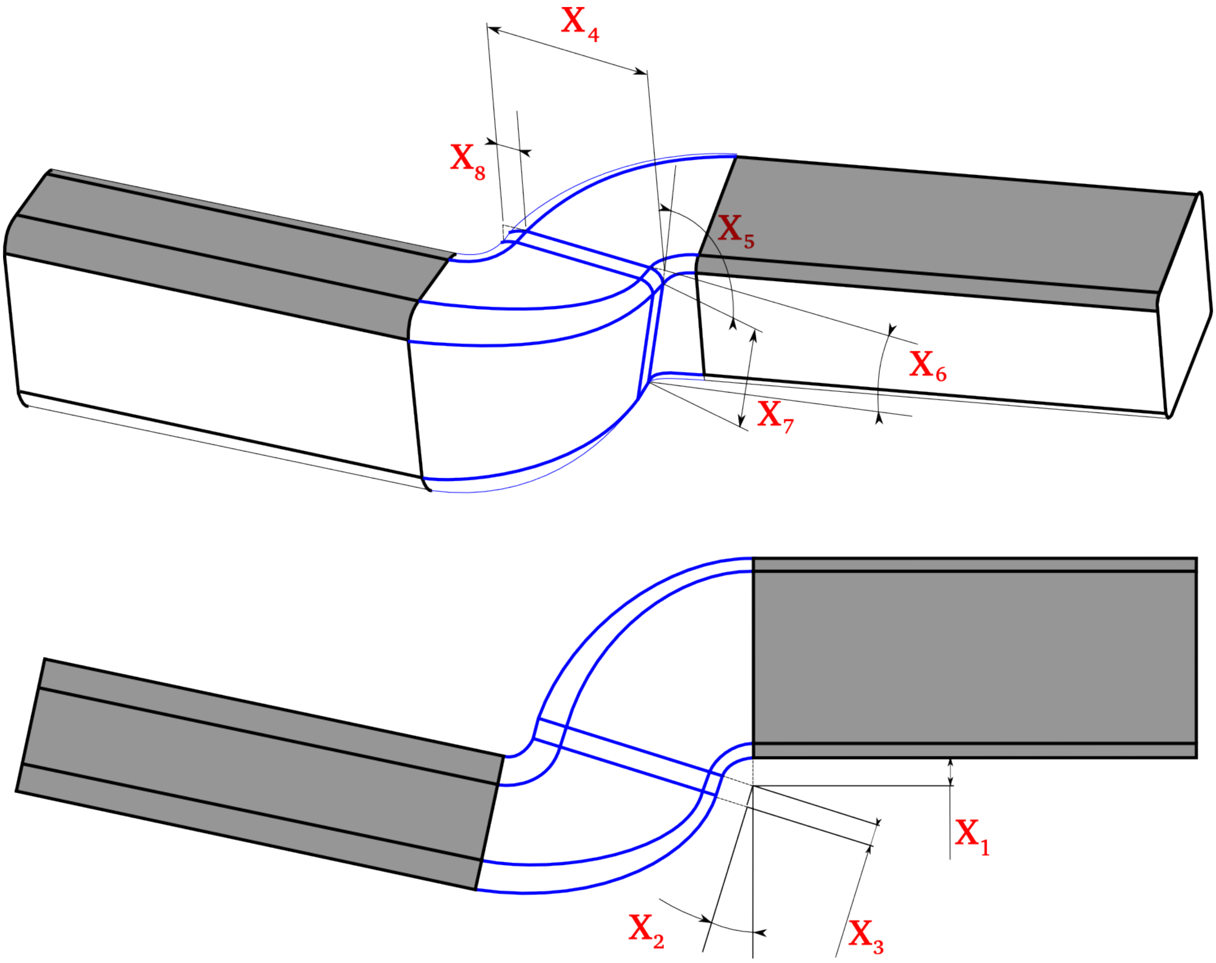


Figure 6: Air-cooling duct for car use. Inlet and outlet ducts are fixed. The elbow shape is defined by 8 parameters called $X_i, i = 1, \ldots, 8$ and B-splines.

The proposed approach is used here to solve a two-criterion optimization problem in subsonic conditions $M_\infty = 0.1$ and transition flow ($Re \approx 2800$). For each evaluation, the compressible Navier-Stokes equations are solved with our in-house CFD code Num3sis. The CAD and the mesh are made with a script style software GMSH [6]. It consists in minimizing the velocity variance as well as the pressure loss computed at a particular duct section, simultaneously, with bound constraints. The two objectives are computed as depiced in Figure 7

In practice, an initial database from $\mathbb{R}^N$ is considered. Then, a Kriging metamodel is generated using this database, for the pressure loss and the velocity variance values. MGDA is applied using each database element as a starting point. Both pressure loss and velocity variance computations are evaluated on the surrogate model. In the same way, gradients are evaluated on the surrogate model. Thus, a non dominated set (with regard to the surrogate model) is obtained. Until a non dominated point is too close to an element of the database (in the search space), its

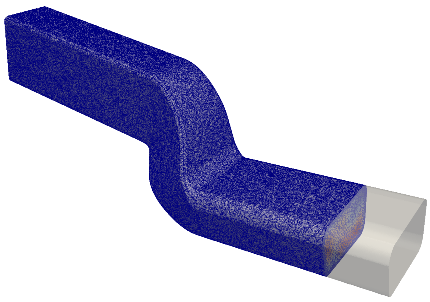
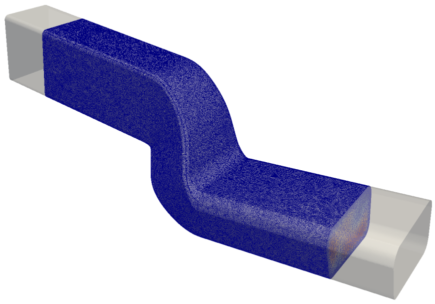
Velocity variance	Pressure loss
• $U_m = \frac{1}{V_{tot}} \sum_{d(cel,\mathcal{P}_o)\leq\varepsilon} u(cel) * Vol(cel)$ • $\sigma^2_{velx} = \frac{1}{V_{tot}} \sum_{d(cel,\mathcal{P}_o)\leq\varepsilon} (U_{mx} - u_x(cel))^2 * Vol(cel)$ • $\sigma^2 = \sigma^2_{velx} + \sigma^2_{vely} + \sigma^2_{velz}$	• $p_k = \lvert P^k_m \rvert$ • $u_k = \lVert U^k_m \rVert$ • $\Delta p = p_i - p_o + \frac{\rho_i u_i^2}{2} - \frac{\rho_o u_o^2}{2}$
	

Figure 7: Objective cost functions computation. The velocity variance is computed in a particular section of the outlet duct, designed in the last line of these tabular. U_m (resp. P_m) is the weighted mean velocity (resp. pressure) in relation to cell volume $Vol(cel)$ and the considered cells total volume V_{tot}. The pressure loss is computed between two particular sections, on of the inlet duct and the second of the outlet duct.

performance is computed using a solver simulation and the initial database is enhanced.

3.3 Numerical results

In the following example, an initial database of 30 points in $\mathbb{R}^8$ is considered. The metamodel assisted MGDA cycle was made 9 times. After each step, the previous database is enhanced by the resulting points of the MGDA applied on the surrogate model, filtered and evaluated with the solver. Finally, a set of 223 points is considered with a non dominated set of 4 points. The total number of evaluations and the best values computed is resumed in Figure 8.

Figure 9 shows step by step the MGDA convergence with exact performance evaluation. For

Number of flow computations	223
Computed nominal velocity variance	0.752
Computed lowest velocity variance	0.477
Computed nominal pressure loss	1.214
Computed lowest pressure loss	0.890

Figure 8: Initial and final statistics (after 9 metamodel-assisted MGDA cycles).

more readability, each step on the Figure represents the points added to the last database. Moreover, on this Figure the non dominated set is indicated for the initial and the final databases for comparison. One can notice a progressive move from the first non dominated set to the Pareto front. For each step illustrated in Figure 9, new non dominated points enhance the global non dominated set. After 9 cycles of the method, the MGDA converged points, on the surrogate model, are close to the results obtained with solver computations at the 9th step. Flow changes during the optimization are shown in the following Figures. The Figure 10 represents velocity

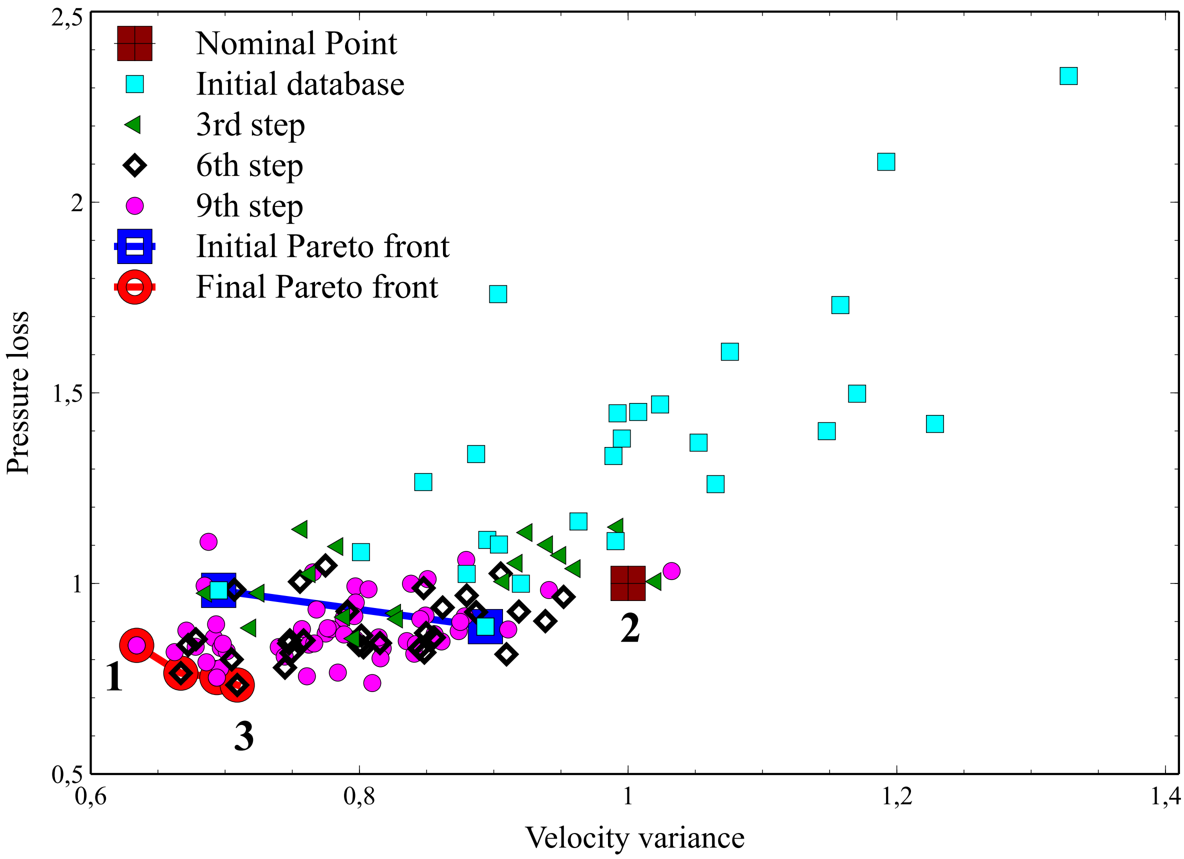


Figure 9: Evolution of the databases for several steps of metamodel assisted MGDA. Comparison between initial and final non dominated sets. Final result with 223 solver simulations. The results are normalized to have the Nominal Point values equal to 1 for both velocity variance and pressure loss.

magnitude on different section of the air-cooling duct. One section is located just after the elbow and the second close to the outlet. The second section indicates the velocity variance is computed. The nominal shape, the lowest velocity variance shape and the lowest pressure loss shape velocity magnitude are represented for comparison. The comparison between the best points computed flows and the nominal flow shows the increase of the longitudinal vortex during the optimization. This raise explains the reduction of velocity variance shown in the outlet of the duct for the two optimal duct shapes.

At the same time, the cross-wise vortex is reduced which explain the pressure loss reduction. This is evident on Figure 10

The reduction of the objectives (minimization of variance velocity and pressure loss simultaneously) requires in both cases an narrower air-cooling duct shapes. This explains the weak difference between the two optimal shapes obtained by the optimization shown on Figure 12 with the nominal shape for comparison.

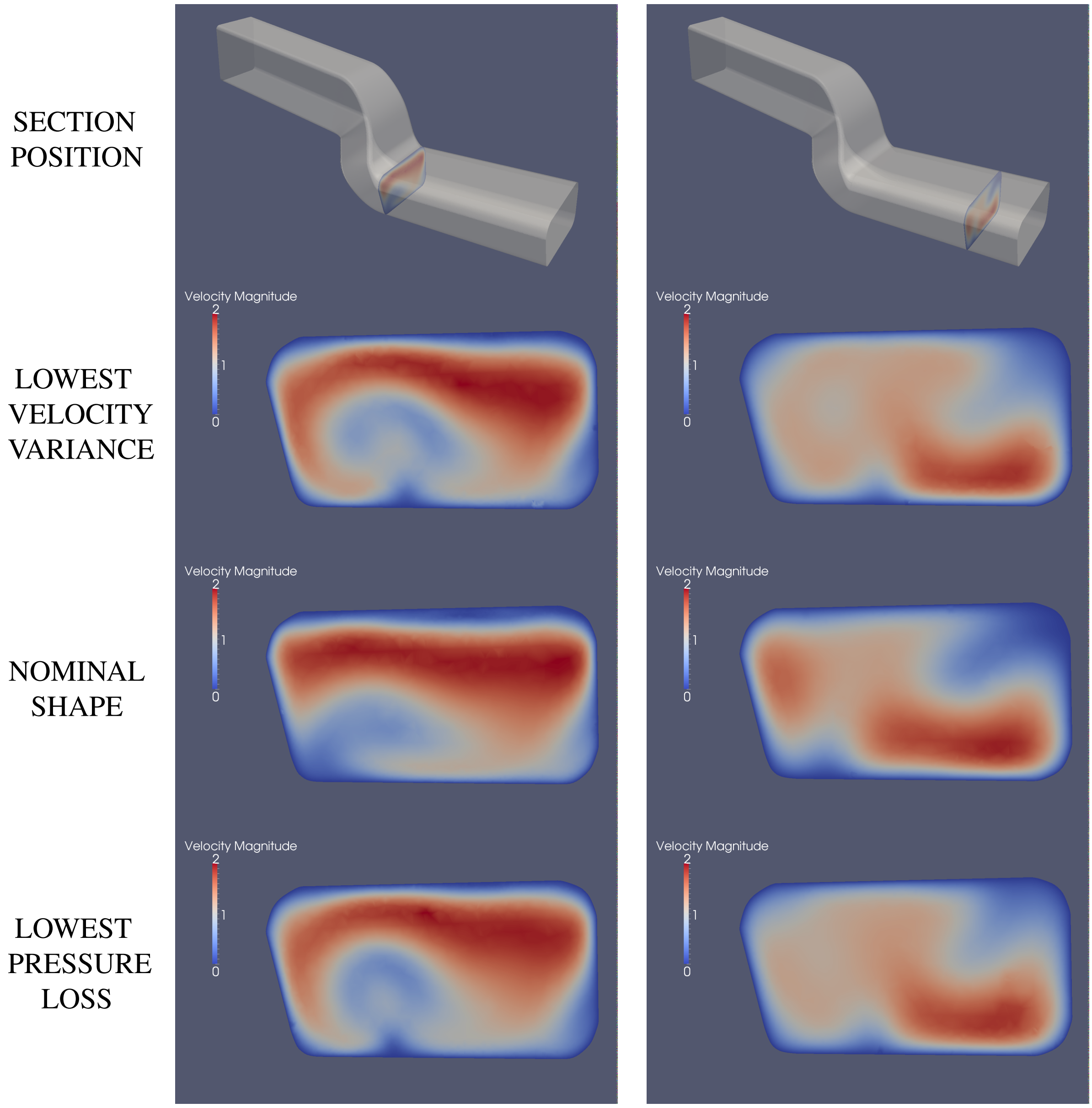


Figure 10: Sections of the air-cooling duct colored according to velocity magnitude. Just after the elbow on the left side and close to the outlet on the right side. The nominal, lowest velocity variance and lowest pressure loss are represented for comparison.

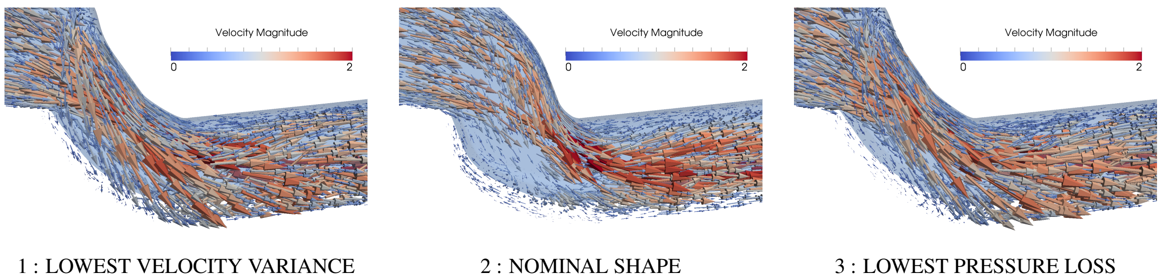


Figure 11: Velocity vectors in 3 cases. The numbers 1, 2 and 3 corresponds to cases indicated on Figure 9.

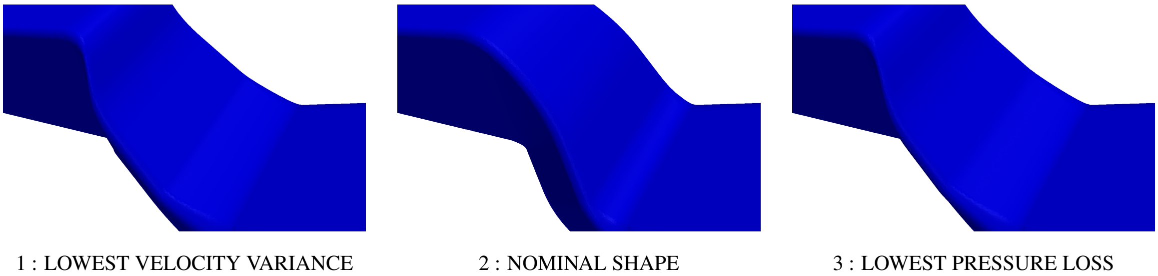


Figure 12: Shapes corresponding to cases 1, 2 and 3.

4 CONCLUSIONS

In this article, we have tested by numerical experiment a recently proposed gradient-based algorithm MGDA [1] for multi-objective optimization. The convergence to Pareto-optimal solutions has been demonstrated in an analytical test case proposed by Fonseca [5] corresponding to a concave Pareto front.

A metamodel assisted MGDA has been developed and successfully tested on a two-objective industrial shape optimization test case. It consists in minimizing variance velocity and pressure loss simultaneously for an air-cooling duct for car using. The result obtained is good with regards to the number of solver evaluations.

In the future, the metamodel-assisted MGDA will be tested on optimization cases with more than two objectives. An ambitious target will be to solve multi-disciplinary optimization problem of concurrent engineering.

REFERENCES

[1] J.A. Désidéri, *MGDA II: A direct method for calculating a descent direction common to several criteria*. INRIA research report, number 7922. Sophia Antipolis, April 2012, http://hal.inria.fr/hal-00685762/PDF/RR-7922.pdf

[2] A. Zerbinati, J.A. Désidéri and R. Duvigneau: *Comparison between MGDA and PAES for multi objective optimization*, INRIA research report, number 7667. Sophia Antipolis June 2011, http://hal.inria.fr/docs/00/60/54/23/PDF/RR-7667.pdf

[3] J. Knowles and D.Corne *Approximating the non-dominated front using the Pareto Archived Evolution Strategy. Evolutionary Computation volume 8, pages 149-172, 1999.*

[4] P.E. Gill, W. Murray and M.H. Wright Practical optimization, Academic Press Inc. [Harcourt Brace Jovanovich Publishers], 1981.

[5] K. Deb, L. Thiele, M. Laumanns and E. Zitzler, *Scalable Test Problems for Evolutionary Multi-Objective Optimization*. TIK-Technical Report number 112, Institut für Technische Informatik und Kommunikationsnetze, ETH Zürich Gloriastrasse 35., ETH-Zentrum, CH-8092, Zürich, Switzerland, 2001.

[6] C. Geuzaine, J.F. Remacle : a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities, http://geuz.org/gmsh/